Estimating Seats–Votes Partisan Advantage

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*This an early draft for review with placeholder citations that need to be formatted properly.*

Abstract

A new class of metrics is defined that measure whether redistricting plans favor one political party or the other: partisan advantage using a seats–votes benchmark. Ten prominent measures of partisan bias are evaluated against a cross section of nearly three dozen past, present, and hypothetical congressional plans. Only (dis)proportionality and the simplified form of the efficiency gap are shown to reliably measure this across a wide range of statewide vote shares. When winner's bonuses are between one and two inclusive, plans are shown to have acceptable bias with respect to the efficiency gap ideal of 2-proportionality. The (dis)proportionality metric is extended to be more robust.

Introduction

This paper proceeds as follows:

1. The many broad criteria for evaluating redistricting plans are identified, and the confusing depth of metrics in the bias subcategory are enumerated.
2. A relatively new concept that measures whether a plan favors one political party or the other – partisan advantage – is extended to use a seats–votes benchmark instead of the original jurisdictional one.
3. Three sets of sample plans are described: the 2011 congressional plans for 11 states using a composite of 2012 election data, the corresponding 2020 plans for those states using a composite of 2016–2020 election results, and a dozen carefully curated hypothetical plans.
4. Ten prominent metrics are calculated for the sample plans: declination (), lopsided outcomes (), mean–median (), seats bias (), votes bias (), geometric seats bias (), global symmetry (), proportional (), the simplified form of the efficiency gap (), and gamma (). Only (dis)proportionality and the efficiency gap are shown to reliably measure seats–votes partisan advantage across a wide range of statewide vote shares.
5. For illustrative purposes, the robustness of the (dis)proportionality measure is increased.

Buckle up!

1. Background

Redistricting technology has been substantially democratized this census cycle. Anyone with access to a web browser can draw block-level maps suitable for submission to redistricting officials[[2]](#footnote-2) and can quickly evaluate plans they and others draw,[[3]](#footnote-3) all free of charge.

People consider many factors when evaluating redistricting plans, including:[[4]](#footnote-4)

* Basic requirements, like whether a plan is complete and contiguous[[5]](#footnote-5)
* Straightforward formulas, like population deviation and measures of compactness[[6]](#footnote-6)
* Much discussed partisan measures of bias & responsiveness[[7]](#footnote-7)
* How much counties, cities, and communities are split by districts[[8]](#footnote-8)
* Whether districts are VRA compliant—a complex analysis that requires experts,[[9]](#footnote-9) and
* Purely qualitative considerations, such as the effects on incumbents and preservation of district cores

Even just under the “bias” subset of the partisan dimension, there are ten prominent metrics that may yield seemingly conflicting results for any given map:[[10]](#footnote-10)

* Declination ()
* Lopsided outcomes ()
* Mean–median ()
* Seats bias ()
* Votes bias ()
* Geometric seats bias ()
* Global symmetry ()
* Proportional ()
* Simplified efficiency gap (),[[11]](#footnote-11) and
* Gamma ()

To complicate matters, different experts focus on different subsets of these metrics: There is no consensus about which of them to use under what circumstances. Moreover, some analysts report simple delegation splits in whole seats, e.g., 10–3 Republicans/Democrats instead of any of these formal metrics.[[12]](#footnote-12)

Despite this unsettled state, some states haven added language like this to their state constitutions:

(a) No apportionment plan or individual district shall be drawn with the intent to **favor or disfavor a political party** or an incumbent; and districts shall not be drawn with the intent or result of denying or abridging the equal opportunity of racial or language minorities to participate in the political process or to diminish their ability to elect representatives of their choice; and districts shall consist of contiguous territory.[[13]](#footnote-13) [emphasis added] --Florida

(d) Districts shall not provide a **disproportionate advantage to any political party**. A disproportionate advantage to a political party shall be determined using accepted measures of partisan fairness.[[14]](#footnote-14) [emphasis added] --Michigan

With tens of thousands of people now able to evaluate hundreds of thousands of redistricting plans,[[15]](#footnote-15) this begs the question “How do you know whether a plan favors one party or the other, gives it an advantage?”

2. Seats–Votes Partisan Advantage

The problem sketched above stems from loose terminology and imprecise definitions:

* People tend to use the terms “bias” and “fairness” interchangeably and sometimes use the term “partisan bias” generically even though it has a specific meaning in the literature.[[16]](#footnote-16)
* Some view what favors or harms a party as a combination of bias and responsiveness, even though the measures are independent.
* People tend to treat all the measures of “bias” as though they measure the same thing – Instead, some metrics measure partisan gerrymandering via packing & cracking,[[17]](#footnote-17) some measure the bias inherent in a state’s political geography,[[18]](#footnote-18) others measure aspects of seats–votes curves to assess partisan symmetry,[[19]](#footnote-19) and still others measure “fairness” relative to some normative standard such as proportionality.[[20]](#footnote-20)

For clarity, I will extend the relatively new concept of partisan advantage[[21]](#footnote-21) to define a class of measures in this last category.

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In our political system dominated by two parties, the number of seats that candidates of the two parties win determines control of Congress and state legislatures: seats won are political currency. Moreover, because district boundaries determine how a state’s votes will likely get translated into seats, single-member redistricting is, in effect, the process of deciding how many seats each party will win.[[22]](#footnote-22) Hence, to gauge whether a proposed or adopted plan will favor one party or the other, you simply compare how votes will *likely* get translated into seats under the plan to some normative ideal for how they *should* get translated into seats in our representative democracy.[[23]](#footnote-23)

Simply put:

*Partisan advantage is the difference between the ideal and actual seat shares.*

Whereas Eguia used a jurisdictional benchmark to account for political geography when he introduced the concept of partisan advantage[[24]](#footnote-24) – call it – I use a seats–votes benchmark which I dub .

Specifically, outside a courtroom the more a plan will likely translate votes into a number of seats that closely matches the ideal seats–votes relationship, the more the plan is politically neutral. The more the map will likely translate votes into seats in a way that significantly deviates from that ideal relationship – whatever it may be! – the more the plan favors one party or the other, the more it confers a seats–votes partisan advantage.

## 2.1. General Formula & Units

To formalize this, call the ideal seats–votes relationship , where and the resulting are the two-party Democratic vote share and seat share, respectively.[[25]](#footnote-25) As you will see below, there are many candidates for this expected seats-votes function, not just proportionality.

Similarly, call the actual the statewide vote share[[26]](#footnote-26) and the resulting seat share and , respectively.

Seats–votes partisan advantage is simply the difference between these two:[[27]](#footnote-27)

(1)

In words, it is the difference between the share of seats that *should* be won given a statewide vote share and the share of seats *actually* won given district vote shares.[[28]](#footnote-28) The unit of measure is a difference of seat shares ().[[29]](#footnote-29) Hence, metrics that don’t compare the difference between actual (or likely) seat shares and some ideal measure some other aspect of a plan. That other quantity may be interesting, but those metrics don’t measure partisan advantage directly.

You can use a composite of prior elections as a proxy for a not-yet-held election to infer a *likely* seats–votes curve and use that to estimate and .[[30]](#footnote-30)

## 2.2. Ideal Seats–Votes Relationships

The concept of is agnostic to what you believe the ideal seats–votes relationship should be, and I won’t advocate a specific one here.[[31]](#footnote-31) Nonetheless, to give you a sense of the breadth of possibilities, I’ll say a few words about some:

* One is simple proportionality, the line on a graph of votes (x-axis) and seats (y-axis). On purely little ‘d’ democratic principles one might say that this the ideal seats–votes relationship:

(2)

Alternatively, in the most real-world terms – only “butts in seats” vote in legislatures! – any deviation from proportionality is an advantage for one party and disadvantage for the other.

* Another is the two times winner’s bonus embedded in the formula for the efficiency gap (). One can argue that this comports better with how single-member districts actually perform in practice.[[32]](#footnote-32) The is the functional form of the simplified formula:

(2)

You will see these first two possibilities reflected in the metrics analyzed in Section 4. You can generalize them to any k-prop line such that , where is the constant responsiveness ().

* You might instead think the non-linear “cube law” is the ideal seats–votes:[[33]](#footnote-33)

(3)

This reduces to:

(4)

* You might argue for a winner-takes-all scenario in which the party that gets most of the votes wins *all* the seats, i.e., for any .
* Or you might advocate that how votes should translate into seats should take into account the underlying political geography of and jurisdictional boundaries within a state.[[34]](#footnote-34)

In short, there are many possible ideal seats–votes relationships.

## 2.3. Super-proportionality

To complete my definition of , I add one minimal constraint:

* Super-proportional outcomes can’t favor the minority party.

In graphical terms shown in Fig. 1, what this means is that a valid measure of can’t classify points D1 or D2 as favoring Republicans or points R1 or R2 as favoring Democrats.

In algebraic terms:

1. If and , a valid measure of will not indicate that the plan favors Republicans.[[35]](#footnote-35)
2. Similarly, if and , a valid measure of will not indicate that the plan favors Democrats.

Plans where a party gets more than half the seats when they receive more than half the votes may or may not be considered biased in favor of that party *depending on which ideal relationship you choose.* More on this below.

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Fig. 1. S(V) space with 2-proportionality ideal (dashed line)

Points D1 & D2 and R1 & R2 are super-proportional outcomes, D3 and R3 are sub-proportional outcomes, and D4 and R4 are anti-majoritarian outcomes.

Note that not all of the seats–votes relationships enumerated in Section 2.2 satisfy this constraint.

With defined, we can sort through the various metrics.

3. Sample Plans

This section presents three groups of partisan “profiles” that I will use to test the ten metrics enumerated above.[[36]](#footnote-36) The 34 plans in them represent a broad cross section of partisan characteristics and, thus, provide a good test suite for these metrics.

## 3.1. Select 2012 Congressional Plans

The first set of maps are the congressional plans studied in (Nagle and Ramsay, 2021). In terms of their typical statewide two-party vote shares, four lean heavily Democratic (California, Illinois, Massachusetts, and Maryland), three lean heavily Republican (South Carolina, Tennessee, and Texas), and four are nearly balanced politically (Colorado, North Carolina, Ohio, and Pennsylvania).

The plans were drawn in 2011 and the profiles use a composite of 2012 election results.[[37]](#footnote-37) The partisan profiles for these plans may be found in the Supplemental Material.[[38]](#footnote-38)

Table

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Table 1 – Metrics for Select 2012 Congressional Plans

The measurements in Table 1 are grouped into five sets of columns:

* The 1st set shows the state, the statewide vote share (), and the corresponding likely seat share ( or simply ).
* The 2nd set shows the overall responsiveness or winner’s bonus for the plan () and the responsiveness at the typical statewide vote share ().
* The 3rd set shows the measures of partisan gerrymandering via packing & cracking: declination (), lopsided outcomes (), and mean–median difference ().
* The 4th set shows the measures of partisan symmetry: seats bias (), votes bias (), geometric seats bias (), and global symmetry ().
* The last set shows the deviation from proportionality (), the efficiency gap (), and gamma ().

All values are denoted as percentages, except which is an angle in degrees and the two measures of responsiveness, and . By convention, positive values indicate Republican advantage and negative values Democratic advantage. The highlighted values will be discussed in Section 4.

The seats–votes curves for the IL and CO plans above are examples of the D1 and D2 super-proportional outcomes in Fig. 1 below, respectively.

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Fig. 2. IL 2012 Congressional

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Fig. 3. CO 2012 Congressional

The NC, OH, and PA seats–votes curves (not shown) are all examples of D4 anti-majoritarian outcomes. Seats­–votes curves for all the plans may be found in the Supplementary Material.

## 3.2. Corresponding 2020 Congressional Plans

The second set of plans are the corresponding 2020 maps for the same states but using a composite of 2016–2020 election.[[39]](#footnote-39) The plans are otherwise the same as the 2012 plans, except in NC and PA where courts redrew them mid-decade.

Table

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Table 2 – Metrics for Select 2020 Congressional Plans

Table 2 above shows the measurements for these metrics. The seats–votes curves for the TX and PA maps below are examples of the R1 super-proportional and D3 sub-proportional outcomes in Fig. 1, respectively.

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Fig. 4. TX 2020 Congressional

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Fig. 5. PA 2020 Congressional

Again, seats–votes curves for all plans may be found in the Supplementary Material.

## 3.3. Warrington’s Hypothetical Plans

Finally, Warrington created a set of 12 hypothetical plans so he could study how well various metrics detected partisan gerrymandering (Warrington, 2019). Each archetypal plan has an associated partisan profile. These are short descriptions of each:[[40]](#footnote-40)

1. A: 1-proportionality – Designed for seats won to track votes received.
2. B: 2-proportionality – Designed such that for all vote shares.
3. C: 3-proportionality – Designed with a responsiveness () of three for all vote shares.
4. D: Sweep – Designed so that Democrats win all the seats, even though the statewide vote share is only 64%, e.g., like Massachusetts congressional plans.
5. E: Competitive – Even though statewide vote share is nearly even (52%) and there are several very competitive races, they all lean slightly towards Democrats.
6. F: Competitive even – Again, the statewide vote share is nearly even (51%) with several competitive districts. Here though none of them fall “in the ‘counterfactual window’ (i.e., between the majority party’s statewide support and 50%”[[41]](#footnote-41)) and they all still lean Democratic.
7. G: Uncompetitive – This profile models an “uncompetitive election as might arise from a bipartisan gerrymander.”[[42]](#footnote-42) The average winning margins for both parties are large. The statewide vote share marginally favors Democrats (52.3%).
8. H: Very uncompetitive – This plan is like the previous example, except that the average winning margins are even more pronounced.
9. I: Cubic – This profile embodies the classic “cube law” seats–votes relationship.
10. J: Anti-majoritarian – Here Democrats get less than half the votes but win more than half the seats.
11. K: Classic – This profile models a classic partisan gerrymander: The statewide vote share is evenly split (50%), but “Republicans win a significant majority through having a number of narrow victories in contrast to their Democratic opponents whose few victories are overwhelming.”[[43]](#footnote-43)
12. L: Inverted – This profile is somewhat complementary to the 2-proportionality example, except the Democratic & Republican vote shares are switched and more extreme.

The essence of each plan is illustrated by rank–votes graphs shown in Fig. 6. The partisan profiles and seats–votes curves for each may be found in the Supplementary Material.

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Fig. 6. Rank–Votes Graphs for Warrington’s Hypothetical Plans

The measurements for these hypothetical profiles are shown in Table 3.

Table

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Table 3 – Measurements for Hypothetical Plans

Warrington evaluated the plans using all-or-nothing first-past-the-post accounting, as opposed to the fractional seat probabilities method that I use, so some of these scenarios may not report as crisply here. To simplify the values, I show the percentages for whole seats in the column.

4. Analysis of Metrics

This section evaluates the ten metrics shown in Tables 1–3 above as potential measures of , using the sample plans described in the previous section.

## 4.1. Measures of Partisan Gerrymandering

The first three metrics measure partisan gerrymandering: declination (), lopsided outcomes (), and mean–median (). While the packing & cracking that they measure is an interesting quantity, none of them measure a difference in seat shares. Hence, they aren’t measures of as I’ve defined it.[[44]](#footnote-44)

These are their detailed definitions.

Given vote shares by district , declination () measures a difference in angles:

(1)

where:

the fractional seat probability for vote share

Lopsided outcomes () measures a difference in *vote* shares:

(2)

Mean–median () also measures a difference in vote shares:

(3)

These measure partisan gerrymandering via packing & cracking but not .[[45]](#footnote-45)

## 4.2. Measures of Partisan Symmetry

The next four metrics measure some aspect of a seats–votes curve: seats bias (), votes bias (), geometric seats bias (), and global symmetry (). Neither votes bias nor global symmetry measure a difference in seat shares, so again they aren’t measures of .

Both seats bias and geometric seats bias *do* measure differences in seat shares, but they sometimes violate the property that super-proportional outcomes can’t favor the minority party. Among many others, two examples are illustrative: the IL 2012 and the TX 2020 plans shown in Fig. 2 and Fig. 4 above.

* In both cases, seats bias gets confounded because the seats–votes curves pass the center point of symmetry[[46]](#footnote-46) on one side of the line of proportionality[[47]](#footnote-47) before crossing over it and reaching the statewide vote share where one party gets a large majority of the votes and an even bigger share of the seats.
* also sends the wrong signal for these two plans, because the vote shares where the counterfactual minority seats–votes curves are evaluated – Republican (red) and Democratic (blue), respectively – are well outside the zone of uncertainty around the statewide vote share.

You can think of these failures as indicative of the *principle of locality* in physics.[[48]](#footnote-48) While seats–votes curves show the likely seat share over all theoretically possible vote shares,[[49]](#footnote-49) only a small range around the typical statewide vote share are likely in practice.[[50]](#footnote-50) Absent some theory and some empirical evidence to support it, there is no reason to believe that any metric that measures a seats–votes curve far away from the likely statewide vote share measures anything related to partisan advantage close to it.

These are their detailed definitions of the partisan symmetry metrics.

Seats bias () measures a difference in seat shares:

(4)

Votes bias () measures a difference in vote shares, vote share required to win 50% of the seats, from the inferred seats–votes curve.

Geometric seats bias () measures a difference in seat shares at the statewide vote share :

(5)

Global symmetry () measures the area of asymmetry between the Democratic (blue) and Republican (red) seats–votes curves – basically the geometric seats bias summed over the entire range of vote shares – normalized by the total seats–votes unit square.

## 4.3. Measures of Seats–Votes Partisan Advantage

That last three metrics share a common underlying functional form:

(6)

where is an actual or idealized value of responsiveness (). They all yield differences in seat shares and – with one slight modification – none violate the principle that super-proportional outcomes can’t favor the minority party.

Proportional () measures a difference between the actual (or likely) seat share () and an ideal seat share that matches the statewide vote share ():

(7)

is zero on the line where , i.e., a responsiveness () of one.

In contrast, the simplified form of the efficiency gap () embodies a constant responsiveness () of two:

(8)

The measurements for both CO plans are highlighted in Tables 1 & 2, as is the value for the 1-proportionality hypothetical plan in Table 3. Here’s why:

The dashed 2-proportionality line in Fig. 1 is where . Above that line, values are negative (indicating Democratic bias), and below that line they are positive (indicating Republican bias). But the formula formalizes the notion that a two-times winner’s bonus is acceptable, so to say that a point just below the 2-proportionality line immediately favors Republicans is to, in some sense, to contradict the essential framework. Hence, I argue that when the winner’s bonus (R) is between one and two inclusive – in the white regions in Fig. 1 – a map is not biased *with respect to the efficiency gap ideal.* I highlight these values to show this.

You can see an example of this in the CO 2012 plan shown in Fig. 3.

The simplified thus modified (or interpreted) is a valid measure of .

Finally, the gamma () metric uses the responsiveness () measured at the statewide vote share:

(9)

The base formula has the analogous “doesn’t acknowledge acceptable bias” issue as , when the responsiveness () is very high. The IL and CO 2012 and IL and TX 2020 plans are examples which is why those values are highlighted.

Technically, an appropriately interpreted would also be a valid measure of , but when the measured responsiveness () is big almost no plan can be judged as favoring the majority party.

5. Increasing Robustness

There are two ways that a valid measure of can be made more robust. One is to incorporate the fact that seats are won in their entirety and the other is to evaluate the measure over a local range that brackets the likely statewide vote share instead of at just that one point. I illustrate both with the proportionality measure of bias ().

## 5.1. Using the Number of Seats Closest to Proportional

compares the likely *fractional* seat share () to the likely statewide vote share (). But you can’t win part of a seat: seats are won all or nothing. A more practical measure I label recognizes that the proportional ideal is better represented as a step function rather than the line and estimates disproportionality relative to the whole number of seats closest to proportional at the statewide vote share.[[51]](#footnote-51)

Simple generally tracks , but there are times when they can diverge somewhat, e.g., both Colorado plans – where B% reports 6.4% and -1.6% deviations instead of -0.2% and -4.2%, respectively – and the North Carolina and Pennsylvania 2020 plans – where B% reports 4.8% and 4.5% deviations instead of 8.1% and 1.7%, respectively.

## 5.2. Evaluating Over a Range Instead of at a Single Point

Proportionality () is evaluated at a single point: the likely statewide vote share (). But the statewide vote share is an *estimate* based on a composite of past elections, and it will ultimately vary somewhat for specific future elections. A new metric that I dub averages B% over range of uncertainty that brackets the statewide vote share.[[52]](#footnote-52) In effect, estimates the area between the Democratic seats–votes curve (blue) and the number of seats closest to the closest-to-proportional step function for the local range of uncertainty (gray in the seats–votes curves).[[53]](#footnote-53)

B% generally tracks , but again there are times when they can diverge somewhat. For example:

* CO 2012 Congressional – The number of seats closest to proportional changes from 3 to 4 in the zone of uncertainty, so it makes a difference where you evaluate (dis)proportionality.
* PA 2020 Congressional – Similarly, the number of seats closest to proportional changes from 9 to 10 in the range around the statewide vote share.

The number of seats closest to proportional varies in the local range around the statewide vote share for eight of the eleven 2012 plans: CA, IL, MA, CO, NC, OH, PA, and TX.

Evaluating using whole seats () and averaging samples over a range around the statewide vote share () increases the robustness of the (dis)proportionality () as a measure of .

Conclusion

Most prominent measures of “bias” don’t measure seats–votes partisan advantage () specifically and/or aren’t reliable for states that are unbalanced politically. Two do measure it directly and are reliable across the full range of statewide vote shares: proportional () and the simplified efficiency gap (). The robustness of these measures can be increased by using a step-function for the number of whole seats closest to proportional and averaging it over the local range of uncertainty around the likely statewide vote share.

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*Citations need to be added throughout the manuscript, and these references need to be formatted properly.*

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[end]

1. I work on Dave’s Redistricting in Seattle, Washington, USA. I would like to thank Gregory Warrington – his early feedback helped me frame these ideas – and Jon Eguia – his patient, gracious feedback helped me develop them. [↑](#footnote-ref-1)
2. Dave’s Redistricting (<https://bit.ly/3drhkdj>) and District Builder (<https://bit.ly/3lF4f4O>). [↑](#footnote-ref-2)
3. Dave’s Redistricting and Plan Score (<https://bit.ly/335rgrd>). While Dave’s Redistricting uses one methodology for partisan metrics (<https://bit.ly/2ZlqVfl>) and Plan Score uses another (<https://bit.ly/3or5IgK>), they both use multiple elections and aggregate census block results up to district-by-district and statewide vote shares. [↑](#footnote-ref-3)
4. Dave’s Redistricting supports all of these directly, except the qualitative considerations, of course. [↑](#footnote-ref-4)
5. Even something as conceptually simple as contiguity is sometimes non-trivial in real life. See the discussion of ‘operational contiguity’ in “Contiguity” (<https://bit.ly/2ZtmB0Q>). [↑](#footnote-ref-5)
6. Dave’s Redistricting computes Reock, Polsby–Popper, and “know it when you see it” compactness (<https://bit.ly/3Gdm8PU>). [↑](#footnote-ref-6)
7. For example, (Tufte 1973), (Grofman 1983), (McDonald et al. 2018), (Stephanopoulos and McGhee 2018), (Wang 2016), (Wang et al. 2019), and (Warrington 2019), (Katz et al. 2020), and (Nagle and Ramsay 2021). [↑](#footnote-ref-7)
8. Appendix 6 of Moon Duchin’s report for the Pennsylvania Supreme Court (<https://bit.ly/3xm1v0S>) and (Wang et al, 2021) are the bases for the county-splitting and community-splitting metrics in Dave’s Redistricting, respectively. [↑](#footnote-ref-8)
9. Dave's Redistricting incorporates some heuristics for evaluating the opportunity for minority representation, based on Moon Duchin's amicus brief submitted for the Bethune-Hill v. VA case (<https://bit.ly/32AkAkz>). [↑](#footnote-ref-9)
10. All of these metrics can be computed from only a statewide vote share and district-by-district vote shares. Dave’s Redistricting reports them all, as well as Jon Eguia’s measure of geographic bias which requires precinct-by-precinct election data (<https://bit.ly/3IpfF6h>). Plan Score calculates the efficiency gap, partisan bias, mean–median, and declination. [↑](#footnote-ref-10)
11. Following (Nagle & Ramsay 2021), I use the simplified form of the efficiency gap which assumes equal turnout (Stephanopoulos & McGhee 2018, 18). This is what Dave’s Redistricting uses as well. [↑](#footnote-ref-11)
12. For example, Dave Wasserman at The Cook Political Report (<https://www.cookpolitical.com/about/staff/david-wasserman>). [↑](#footnote-ref-12)
13. Florida State Constitution (<https://bit.ly/3y2d0dX>) [↑](#footnote-ref-13)
14. Michigan State Constitution (<https://bit.ly/3ExfgfJ>) [↑](#footnote-ref-14)
15. Dave’s Redistricting alone has this level of usage. [↑](#footnote-ref-15)
16. King’s geometric seats bias () measure. [↑](#footnote-ref-16)
17. Declination () does provably well on this (Warrington, 2019). [↑](#footnote-ref-17)
18. See (Eguia, 2021). [↑](#footnote-ref-18)
19. The principle that a plan should treat the two parties equally. See <https://bit.ly/3EUb3Dd>. [↑](#footnote-ref-19)
20. These apples-and-oranges categories cannot be meaningfully compared or combined. [↑](#footnote-ref-20)
21. (Eguia 2021) introduces this concept using a jurisdictional benchmark. [↑](#footnote-ref-21)
22. Some districts may be competitive and “flip” between parties, of course, but that is increasingly rare, and the general point holds. [↑](#footnote-ref-22)
23. One’s notion of how votes *should* translate into seats won does not need to depend solely on an inferred seat-votes curve. It could depend on other factors such as the geographic or racial distribution of votes shares or of turnout across the state. I restrict my focus here to those that be computed from a statewide vote share and district-by-district vote shares. [↑](#footnote-ref-23)
24. (Eguia 2021) [↑](#footnote-ref-24)
25. I use two-party Democratic vote shares by convention. Two-party Replication vote shares are simply . [↑](#footnote-ref-25)
26. John Nagle has called this <V> in the past. [↑](#footnote-ref-26)
27. This is essentially the same formula as (Eguia 2021, 7) with different notation. [↑](#footnote-ref-27)
28. Jon Eguia has pointed out: “This notion of the ideal number of seats requires that each vote count equally, regardless of who casts it and where. There are other notions of fairness leading to other ‘ideals’ about seat outcomes. For instance, under the VRA, or under the Northern Ireland Good Friday Accords, *who* casts each vote matters when determining whether an outcome is fair, since group representation of distinct ethnic groups is deemed essential to fairness. Similarly, under much of the US Democratic tradition – including most first State constitutions -- *where* a vote was cast also mattered, since representation to state assemblies was by county. (Where a vote is cast is still essential to the US Senate and the US Electoral College, of course.) Finally, SCOTUS has explicitly rejected this notion of fairness.” Since my focus is on advantage *in the legislative body* as opposed to fairness though, a seats–votes ideal is the right benchmark. [↑](#footnote-ref-28)
29. In programming terms, we call this the return type of the function. [↑](#footnote-ref-29)
30. Dave’s Redistricting uses John Nagle’s methodology of fractional seat probabilities described in <https://lipid.phys.cmu.edu/nagle/Technical/FractionalSeats2.pdf> and a seats–votes curve inferred using proportional shift described in <https://lipid.phys.cmu.edu/nagle/Technical/2019-04-19%20-%20Measuring%20Redistricting%20Bias%20&%20Responsiveness.pdf>. Plan Score takes a directionally similar approach different in the details. [↑](#footnote-ref-30)
31. I’m not making the argument that proportionality is the ideal seats–votes relationship! [↑](#footnote-ref-31)
32. (Stephanopoulos & McGhee, 2018): “a fourth parameter is *empirical correspondence*. That is, the electoral ideal implied by a metric should not be too different from the American historical norm. Otherwise, the measure would imply that most American plans have been gerrymanders—and its adoption would be so disruptive as to be infeasible.” [↑](#footnote-ref-32)
33. See (Grofman, 1983) and (Tufte, 1973). Alternatively, the power of three can be replaced by any constant. [↑](#footnote-ref-33)
34. As (Eguia 2021) does. [↑](#footnote-ref-34)
35. To make formulas easier to write, I express percentages in the body text as [0–1] fractions. Except as noted, in tables and figures, I show them as percentages. [↑](#footnote-ref-35)
36. A partisan profile consists of a statewide vote share and district-by-district vote shares, using Democratic two-party votes by convention. [↑](#footnote-ref-36)
37. I call them “2012” plans hereafter. [↑](#footnote-ref-37)
38. Supplementary Material is available in this GitHub repository: <https://github.com/dra2020/bias-irl>. [↑](#footnote-ref-38)
39. These maps can be found in the Official Maps collection of Dave’s Redistricting App: <https://davesredistricting.org/>. The composite is described in “Election Composites” (<http://bit.ly/2SeQoDV>). The specific elections used in each state’s composite are documented in the Supplementary Material. [↑](#footnote-ref-39)
40. More details may be found in Warrington, 2019. [↑](#footnote-ref-40)
41. Warrington, 2019. [↑](#footnote-ref-41)
42. Warrington, 2019. [↑](#footnote-ref-42)
43. Warrington, 2019. [↑](#footnote-ref-43)
44. Even though their units of measure invalidate them as measures of , their measurements sometimes also violate the property that super-proportional outcomes can’t favor the minority party, e.g., suggesting that the IL 2012 plan favors Republicans. [↑](#footnote-ref-44)
45. See (Warrington, 2019) for a discussion of how well each of these metrics measure packing & cracking. [↑](#footnote-ref-45)
46. The **point of symmetry** is the point in the center of the seats–votes space. All symmetric seats–votes curves pass through this point. [↑](#footnote-ref-46)
47. The **line of proportionality** is line through the point of symmetry where . [↑](#footnote-ref-47)
48. <https://en.wikipedia.org/wiki/Principle_of_locality> [↑](#footnote-ref-48)
49. Because statewide vote shares tend to not fall much outside the range [0.4, 0.6], I only infer the points of the seats–votes curve for the range [0.25, 0.75]. [↑](#footnote-ref-49)
50. I consider the **zone of uncertainty** around the statewide vote share to be a 5% range that brackets it, because the average uncertainty for the seats–votes curves in (Nagle and Ramsay, 2021) was roughly 2%. [↑](#footnote-ref-50)
51. This is the measure of (dis)proportionality used in DRA in the *Proportionality* section of the *Analyze* tab. It is implemented in <https://github.com/dra2020/dra-analytics>. [↑](#footnote-ref-51)
52. This is implemented in <https://github.com/dra2020/dra-analytics> as an “experimental” measure, and you can have it shown on the *Advanced* tab in Dave’s Redistricting on request. [↑](#footnote-ref-52)
53. Remember, I define “local” to be the 5% range that brackets the statewide vote share, i.e., +/– 2.5%. [↑](#footnote-ref-53)